Quantum theory is incompatible with relativity: A new proof beyond Bell's theorem and a test of unitary quantum theories

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Abstract

It has been debated whether quantum mechanics and special relativity are incompatible and whether there is a preferred Lorentz frame if they are incompatible. Bell's theorem is an important cornerstone, but it does not give us a definite positive answer due to the existence of supplementary assumptions or theoretical loopholes; there are unitary quantum theories which evade Bell's theorem and claim that they are compatible with special relativity.

In this paper, I address the important issue of whether unitary quantum theories are compatible with special relativity. I propose a new Gedankenexperiment, a variant of the EPR-Bohm experiment with a superobserver who can undo a measurement. In this experiment, there is a stronger correlation (between the results of two spacelike separated measurements) than the correlation investigated in Bell's theorem. Based on an analysis of the correlations in different Lorentz frames, I prove that unitary single-world theories are incompatible with special relativity, and in order to avoid the incompatibility, there must exist a preferred Lorentz frame in these theories. Moreover, I argue that the incompatibility proof also applies to a proper version of the many-worlds interpretation of quantum mechanics. This closes the major theoretical loopholes of Bell's theorem, including relationalism, retrocausality, and superdeterminism.

Finally, I argue that the stronger correlation found in the Gedankenexperiment cannot be explained by retrocausal processes or even the common causes in the past, but only be explained by nonlocal processes or actions at a distance. This provides a test of unitary quantum theories, as well as a further support for the new incompatibility proof beyond Bell's theorem.

1 Introduction

It has been debated whether quantum mechanics and special relativity are incompatible and whether there is a preferred Lorentz frame if they are incompatible. In 1964, based on the Einstein-Podolsky-Rosen (EPR) argument [1], Bell derived an important result that was later called Bell's theorem [2]. It states that certain predictions of quantum mechanics cannot be accounted for by a local realistic theory, and thus strongly suggests that quantum mechanics and special relativity are incompatible. More than 50 years later [3, 4], although the loopholes in Bell test experiments have been closed with advances in quantum technology [5-7], the theoretical loopholes of Bell's theorem are still there; there are unitary quantum theories (without wave function collapse) which evade Bell's theorem and claim that they are compatible with special relativity, such as the many-worlds interpretation of quantum mechanics (MWI) [8, 9], retrocausal theories [10-12], relational quantum theories [13-16], and superdeterminism [17].

The main purpose of this paper is to close these theoretical loopholes of Bell's theorem, including the superdeterminism loophole. It will be argued that a unitary quantum theory is incompatible with special relativity, and in order to avoid the incompatibility, there must exist a preferred Lorentz frame in the theory. The rest of this paper is organized as follows. In Section 2, I will first briefly analyze collapse theories of quantum mechanics to which Bell's theorem applies. It is pointed out that in these theories there is a preferred Lorentz frame in which the collapse of the wave function is simultaneous in different regions of space. From Section 3 to Section 5, I will analyze unitary single-world quantum theories that assume the result of each measurement is unique. In Section 3, I will propose a new Gedankenexperiment, a variant of the EPR-Bohm experiment with a superobserver who can undo a measurement. In this Gedankenexperiment, a stronger correlation (between the results of two spacelike separated measurements) than the correlation investigated in Bell's theorem is found, which is the key to derive a stronger result than Bell's theorem. In Section 4, I will give a proof of the incompatibility between unitary single-world theories and special relativity based on an analysis of the stronger correlations in different Lorentz frames in the Gedankenexperiment. It is argued that in order to avoid the incompatibility, there must exist a preferred Lorentz frame in a unitary quantum theory.

In Section 5, I will analyze the implications of this new result for unitary single-world theories. If the result holds true in a theory but the theory cannot explain the stronger correlation in the Gedankenexperiment and further accommodate a preferred Lorentz frame, then the theory will be inconsistent or even wrong. First, it is pointed out that the incompatibility result is valid in non-relational quantum theories such as superdeterminism and retrocausal theories. Moreover, a relationism loophole of the proof is closed;

it is argued that relational quantum theories cannot evade the incompatibility result either. Next, it is argued that (local) superdeterminism, which may explain the correlations in Bell's theorem in principle, cannot explain the stronger correlation in the Gedankenexperiment. Third, it is argued that retrocausal theories cannot explain the stronger correlation in the Gedankenexperiment either. Fourth, it is shown that the correlation can be naturally explained by nonlocal processes or actions at a distance in a nonlocal theory such as Bohm's theory. Sixth, other non-ψ-ontic quantum theories such as consistent histories and QBism are briefly analyzed. It is argued that if the superobserver's reset operation in the Gedankenexperiment is valid in these theories, then the incompatibility proof will apply to them. In Section 6, I will analyze the many-worlds interpretation of quantum mechanics (MWI) that rejects unique measurement results. It is argued that the incompatibility proof also applies to a proper version of MWI. Conclusions are given in the last section.

2 Collapse theories of quantum mechanics

Consider a usual EPR-Bohm experiment. There are two observers Alice and Bob who are in their separate laboratories and share an EPR pair of spin 1/2 particles in the spin singlet state:

$$\frac{1}{\sqrt{2}}(\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2). \tag{1}$$

Alice measures the spin of particle 1 at angle a, and Bob measures the spin of particle 2 at angle b. These two measurements can be spacelike separation. Each measurement result is $+1$ or -1, corresponding to spin up or spin down. Then we can calculate the probabilistic correlation function $E(a,b)$ for Alice's and Bob's measurement results according to the Born rule, which is $E(a,b) = -cos(a-b)$. In particular, in the EPR anti-correlation case of $b = a$, we have $E(a,b) = -1$, which means that when Alice's result is $+1$, Bob's result is -1, and vice versa.

There are two categories of quantum theories which can explain the EPR correlations, unitary quantum theories without wave function collapse and collapse theories [18]. I will first consider collapse theories briefly and then consider unitary quantum theories in more detail. In collapse theories, the collapse of the wave function is simultaneous in different regions of space in a preferred Lorentz frame. Suppose Alice first measures particle 1 in the above example. Then, in the preferred Lorentz frame, Alice's measurement on particle 1 instantaneously influences the state of particle 2 (as well as the state of particle 1)[1], and the dynamical collapse of the entangled state

[1]Here I omit the finite collapse time, which can be made arbitrarily short in principle.

of particles 1 and 2 due to Alice's measurement happens simultaneously in the regions of particles 1 and 2, no matter how far apart they are.

Due to the existence of a preferred Lorentz frame, collapse theories are incompatible with special relativity (see also [19]).[2] This is reflected in two aspects. The first aspect is the violation of the Lorentz invariance of the relation of temporal precedence (i.e. causes always preceding their effects temporally). In the preferred Lorentz frame, causes always precede their effects temporally. For example, in this frame, Alice's measurement as a cause temporally precede the change of the state of particle 2 as its effect. But in some other Lorentz frames, due to the Lorentz transformations, causes temporally follow their effects, e.g. Alice's measurement as a cause temporally follows the change of the state of particle 2 as its effect.

The second aspect is the violation of the Lorentz invariance of the Born rule. In the preferred Lorentz frame, since the collapse of the wave function is simultaneous in different regions of space, the integral of the modulus squared of the wave function defined at a given instant in the whole space is always one, and the Born rule is always satisfied. However, in all other Lorentz frames, since the collapse of the wave function is not simultaneous in different regions of space, the integral of the modulus squared of the wave function defined at a given instant in the whole space is not one. As a result, the total probability as given by the integral is not one, and the Born rule is violated. In other words, the modulus squared of the wave function defined at a given instant cannot be interpreted as a probability as required by the Born rule. Note that in these Lorentz frames, the integral of the modulus squared of the wave function in different regions at different appropriate instants is still one, and just because the collapse of the wave function is not simultaneous in different regions of space either, the total probability for collapse results or measurement results is still one, and thus the violation of the Born rule cannot be measured (see Gao, 2017).

3 An EPR-Bohm experiment with superobserver

Now let's consider unitary quantum theories. In these theories, Alice's measurement on particle 1 does not collapse the entangled state of particles 1 and 2, and in particular, it does not collapse the state of particle 2 which may be far away from particle 1. Thus it seems that unitary quantum theories can be compatible with special relativity. As we will see, however, this is not the case. The key is to notice that in unitary quantum theories, Alice's measurement on particle 1 can be undone *locally* without interacting with particle 2 and Bob so that the wave function of all systems, including

[2] This raises an interesting issue of whether the relativistic versions of collapse theories proposed so far are fully relativistic [20-22]. For a recent review see [23].

particle 2, can be reset to the initial one,[3] and this permits the existence of a stronger correlation between Alice's and Bob's measurement results, which can reveal the incompatibility between unitary quantum theories and special relativity.

There are two categories of unitary quantum theories: single-world theories and many-worlds theories. I will first analyze the single-world theories in which the result of each measurement is unique. For the purpose of convenience, I will usually call unitary single-world theories unitary quantum theories in the following, unless stated otherwise.

Consider a variant of the above EPR-Bohm experiment in which there is an additional superobserver in Alice's laboratory who can undo her measurement.[4] First, suppose in the laboratory frame (in which Alice's and Bob's laboratories are at rest), Alice first measures the spin of particle 1 at angle a and obtains her result, then the superobserver undoes Alice's measurement (which restores the states of Alice and the particles to their initial states), and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then the superobserver undoes Alice's second measurement, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at angle $b = a$. According to the Born rule, the probability distribution of Alice's results is $P(+1) = 1/2$ and $P(-1) = 1/2$.

Next, suppose in the laboratory frame Bob first measures the spin of particle 2 at angle $b = a$, and then Alice measures the spin of particle 1 at angle a and obtains her result, and then the superobserver undoes Alice's measurement, and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then the superobserver undoes Alice's second measurement, and this process in Alice's side repeats a large number of times. In this case, according to the Born rule, the probability distribution of Alice's results is $P(+1) = 1$ and $P(-1) = 0$ (when Bob's result is -1) or $P(+1) = 0$ and $P(-1) = 1$ (when Bob's result is $+1$).

It can be seen that the results of Alice's measurements are correlated with Bob's measurement, independently of whether Bob's measurement is spacelike separated from Alice's measurements. When Bob does not make a measurement before Alice's measurements, Alice will obtain two different results, spin up and spin down, with roughly equal frequency, while when Bob makes a measurement before Alice's measurements, Alice will always

[3] In collapse theories, however, Alice's measurement on particle 1 cannot be undone locally, since it will collapse the entangled state of particles 1 and 2 into a product state of the two particles, each of which is in a definite spin state. This point is emphasized by Dieks [16].

[4] There have been some discussions about superobservers and similar Gedankenexperiments in the literature [24-30]. Note that for the purpose of my analysis the superobserver only needs to restore the wave function of relevant systems. Even if there exist other states of reality or hidden variables of these systems and they are changed by Alice's measurement, the superobserver needs not restore them.

obtain the same result, either spin up or spin down. This is a stronger correlation than the correlation investigated in Bell's theorem. In fact, it is the strongest correlation in a quantum theory where the no-signalling theorem is valid; the correlation is only weaker than superluminal signalling. Note that at the end of these experiments, all of Alice's measurement results are erased by the superobserver. Thus, the statistics of Alice's results can only be calculated from a theory, and it cannot be found by experiments. This is consistent with the no-signaling theorem; the correlation cannot be used for superluminal signalling.

4 A proof of the existence of a preferred Lorentz frame

Notwithstanding its consistency with the no-signaling theorem, a unitary quantum theory is incompatible with special relativity. I will give a rigorous proof in this section.

Suppose in the laboratory frame Alice and the superobserver first make their series of measurements and reset operations and then Bob makes his measurement. Then Alice will obtain two different results, spin up and spin down, with roughly equal frequency. When Bob's measurement is spacelike separated from Alice's measurements, the following temporal order of events in another inertial frame is permitted by special relativity.[5] In this frame, Bob first makes his measurement, and then Alice and the superobserver make their series of measurements and reset operations. Then Alice will obtain the same result each time, either spin up or spin down. Since the results of the same measurement observed in two Lorentz frames should be the same, there is a contradiction.[6]

One might have an immediate objection here. Even though Alice can repeat her measurement many times, as noted above, she will not be able to remember or report the statistics of these results in a unitary quantum theory. Thus one may insist that the statistics of Alice's results does not exist. If there is no such a thing, then the above analysis based on it will be invalid. This objection can be answered. That Alice cannot remember or report the statistics of her results only means an impossibility of testing certain predictions of a theory at the empirical level,[7] while what I consider here is whether the predictions of two theories are compatible at the theoretical level. After all, the statistics of the results of Alice's repeated

[5]Note that when the distance between Alice's and Bob's laboratories is very large and the duration between Alice's measurements and Bob's measurement is very short, the relative velocity between this inertial frame and the laboratory frame may be close to zero.

[6]I will discuss Dieks' perspectivalism [16] later, which denies that measurement results are frame-independent.

[7]Admittedly, this does raise an interesting issue for philosophy of science.

measurements in each Lorentz frame can be properly defined and also precisely predicted by a unitary quantum theory and special relativity.[8] The result I have derived above is just that the combination of these two theories will lead to a contradiction when considering their predictions for the statistics of Alice's results in different Lorentz frames.

In fact, a similar contradiction can also be derived without resorting to the statistics of the results of Alice's repeated measurements. Suppose in the laboratory frame, Alice first measures the spin of particle 1 at angle a and obtains her result, then the superobserver undoes Alice's measurement, which restores the states of Alice and the particles to their initial states, and finally Bob measures the spin of particle 2 at angle $b = a$ and obtains his result. When Bob's measurement is spacelike separated from the superobserver's reset operation, the following temporal order of events in another Lorentz frame is permitted by special relativity. In this frame, Alice first measures the spin of particle 1 at angle a and obtains her result, then Bob measures the spin of particle 2 at angle $b = a$ and obtains his result, and finally the superobserver undoes Alice's measurement. According to the Born rule, in this frame, when the result of Alice's measurement is $+1$, the result of Bob's measurement must be -1 with certainty. On the other hand, in the laboratory frame, when the result of Alice's measurement is $+1$, the result of Bob's measurement cannot be -1 with certainty. The reason is that if the result of Bob's measurement is -1 with certainty, then if Alice makes her second measurement her result will be $+1$ with certainty, which further means that if Alice and the superobserver repeat their measurements and reset operations a large number of times, Alice's results will be all $+1$, which violates the Born rule. Thus there is a contradiction.

This contradiction can be seen more clearly when considering measurements on an ensemble of the EPR pairs of spin $1/2$ particles in the spin singlet state. Then, when the result of Alice's measurement is $+1$, the result of Bob's measurement may be $+1$ sometimes in the laboratory frame (where Alice's measurement is undone before Bob's measurement), while in another Lorentz frame where Alice's measurement is undone after Bob's measurement, the result of Bob's measurement is always -1. Note again that the results of the same measurement observed in two inertial frames are the same. Thus we have a contradiction.

The contradiction is more obvious when assuming Alice's and Bob's measurement operations are independent of each other (e.g. no superdeterminism). In this case, when Alice's measurement is undone before Bob's measurement, Alice's and Bob's measurements are equivalent to two independent measurements on two spin singlet states, and thus their measure-

[8] In addition, it is worth pointing out that whether the statistics of Alice's results exists or not (when she cannot remember or report the statistics) does not depend on any quantum theory, and it is purely a classical issue.

ment results will be independent of each other. This means that when the result of Alice's measurement is $+1$, the result of Bob's measurement may be $+1$ or -1 with the same probability $1/2$ in the laboratory frame, while in another Lorentz frame where Alice's measurement is undone after Bob's measurement, the result of Bob's measurement is always -1 (see also [16]). Thus the contradiction will appear with probability $1/2$.

The existence of the above contradiction means that a unitary quantum theory is incompatible with special relativity. Moreover, a unitary quantum theory can be valid only in a preferred Lorentz frame. For if there existed two Lorentz frames in which a unitary quantum theory is valid, then we could arrange the temporal order of Alice's and Bob's measurements so that the predictions of the theory in the two Lorentz frames contradict each other as shown above.[9] This means that in a unitary quantum theory there must exist a preferred Lorentz frame, in which the temporal order of events is real and the predictions of the theory are always true, while in other Lorentz frames the temporal order of events is not real and the predictions of the theory cannot be always true.[10]

5 A test of unitary quantum theories

It is well known that there are unitary single-world theories which evade Bell's theorem. Examples include relational quantum theories, such as relational quantum mechanics [13, 14] and perspectivalism [15, 16], and non-relational quantum theories, such as retrocausal theories [10-12] and superdeterminism [17]. It is widely thought that these theories can explain the Bell inequality-violating correlations predicted by quantum mechanics and are also compatible with special relativity. In this section, I will analyze the implications of the above incompatibility result on these theories. There are two possible implications. One is that although a theory can evade Bell's theorem, it cannot evade the above result, and thus the popular view that the theory is compatible with special relativity is wrong. The other is that the above result holds true in a theory but the theory cannot explain the stronger correlation (that is used to prove the result) and further accommodate a preferred Lorentz frame, and thus the theory is inconsistent or even wrong.

[9] Here the invariance of the one-way speed of light or standard synchrony is assumed as usual. If one adopts the convention of nonstandard synchrony that restores the absoluteness of simultaneity (see, e.g. [31], chap. 9), then a unitary quantum theory can be valid in all Lorentz frames. But the one-way speed of light will be not isotropic in all but one Lorentz frame, and thus the non-invariance of the one-way speed of light will also single out a preferred Lorentz frame, in which the one-way speed of light is isotropic.

[10] Similar results have also been obtained in [32, 33].

5.1 The proof reconsidered: closing the relationism loophole

In non-relational quantum theories such as retrocausal theories and superdeterminism, physical facts such as measurement results are the same relative to all physical systems including all Lorentz frames. In particular, the results of the same measurement observed in two Lorentz frames are the same. Thus, the above incompatibility proof is valid in these theories. In a Lorentz frame in which Alice and the superobserver first make their series of measurements and reset operations and then Bob makes his measurement, Alice will obtain two different results, spin up and spin down, with roughly equal frequency according to the Born rule. When Bob's measurement is space-like separated from Alice's measurements, special relativity permits the existence of another Lorentz frame in which Bob first makes his measurement and then Alice and the superobserver make their series of measurements and reset operations. In this frame, Alice will obtain the same result each time, either spin up or spin down, according to the Born rule. Since the results of Alice's measurement observed in these two Lorentz frames are the same, there is a contradiction. The existence of this contradiction means that retrocausal theories and superdeterministic theories, if they are consistent with these predictions of quantum mechanics, are also incompatible with special relativity, and in these theories there should also exist a preferred Lorentz frame, only in which the temporal order of events is real and the predictions of these theories are always true.

In relational quantum mechanics [13, 14], physical facts or events such as measurement results may be different relative to different physical systems. But this theory does not explicitly claim that measurement results are frame-dependent. For example, in this theory, Lorentz transformations does not translate a measurement result such as x-spin up in one Lorentz frame into another result such as x-spin down in another Lorentz frame, and the results of the same measurement observed in two Lorentz frames are always the same. Thus, relational quantum mechanics cannot avoid the incompatibility result either. In other words, this theory is also incompatible with special relativity, and there should also exist a preferred Lorentz frame in the theory.

Finally, perspectivalism does assume that measurement results are frame-dependent [15, 16] On this view, Lorentz transformations can translate a measurement result such as x-spin up in one Lorentz frame into another result such as x-spin down in another Lorentz frame. Thus, at first sight, perspectivalism, different from relational quantum mechanics, may avoid the incompatibility result. Indeed, this seems to be the only way to permit that single-world quantum mechanics and special relativity are compatible. However, this view seems too radical. In Healey's words, physical facts such as measurement results are no longer objective on this view [30]. As Dieks also admitted, "Accepting that physical properties are not monadic and locally defined, but rather perspectival, relational or hyperplane dependent is a huge step away from everyday experience and from the intuitions that served us so well in nonquantum physics." [16]

Moreover, a further analysis seems to indicate that even perspectivalism cannot avoid the incompatibility between quantum mechanics and special relativity. In order to avoid the incompatibility, the Lorentz transformations need to translate the distribution of Alice's results in one frame into the distribution of her results in another frame, such as translating two different results, x-spin up and x-spin down, with roughly equal frequency into the same result each time, either x-spin up or x-spin down. It is obvious that a definite translation rule cannot do this, since any definite translation rule always translate the same result each time into the same result each time. On the other hand, if the translation rule is random, it is hard to imagine how it can translate two different results with roughly equal frequency into the same result each time.

Even if perspectivalism can avoid the incompatibility result by assuming that measurement results are frame-dependent, this view alone cannot explain the stronger correlation between the results of Alice's measurements and Bob's measurement choice in one Lorentz frame (when these two measurements are spacelike separated). Moreover, the correlation also poses further difficulties for the Lorentz transformations of Alice's results discussed above, since it requires that the transformations should depend on Bob's measurement choice; when Bob measures the spin of particle 2 at angle $b \neq a$, the Lorentz transformations of Alice's results should be different from those for the anti-correlation case of $b = a$.

The stronger correlation between the results of Alice's measurements and Bob's measurement choice also poses a challenge for non-relational quantum theories such as superdeterminism and retrocausal theories. Can these theories explain the correlation? I will try to answer this question in the next sections.

5.2 Superdeterminism

Let's first see superdeterminism. Superdeterminism admits the relation of temporal precedence but does not use it to explain the correlation between the results of Alice's measurements and Bob's measurement choice in a Lorentz frame. Rather, it resorts to the common cause of these two measurements in the past to explain the correlation. Then, since the dynamics of a superdeterministic theory is supposed to be local (in order to avoid the nonlocality implication of Bell's theorem) [17], even if the correlation depends on the time difference between these two measurements, the dependence relation is arguably continuous, which means that when the time difference between these two measurements is arbitrarily small, the difference between the correlations is also arbitrarily small. In other words, the correlation between the results of Alice's measurements and Bob's measurement choice does not depend on the temporal order of Alice's measurements and Bob's measurement according to superdeterminism. But this contradicts the predictions of quantum mechanics. No matter how small the time difference between Alice's measurements and Bob's measurement, the correlation for the case of Alice's measurements preceding Bob's measurement and the correlation for the case of Bob's measurement preceding Alice's measurements are always greatly different; when Alice's measurements precedes Bob's measurement, Alice will obtain two different results, spin up and spin down, with roughly equal frequency, while when Bob's measurement precedes Alice's measurements, Alice will obtain the same result each time, either spin up or spin down.

Besides the difficulty to explain the correlation, a more serious issue of a local superdeterministic theory (that evades Bell's theorem) is that it does not violate the equivalence of all Lorentz frames and thus cannot accommodate a preferred Lorentz frame. This means that the existence of a preferred Lorentz frame will exclude these theories. In fact, if superdeterminism cannot avoid the incompatibility between quantum mechanics and special relativity, then it seems that there will be no good motivation to pursue such a radical view.

If superdeterminism is untenable, and the correlation between the results of Alice's measurements and Bob's measurement choice does not result from a common cause in the past, then the correlation must originate from certain causal influences of Bob's side on Alice's side. In this case, there are only two ways to explain the correlation: assuming the relation of temporal precedence and resorting to nonlocal processes or actions at a distance, or rejecting the relation of temporal precedence and resorting to local retrocausal processes.

5.3 Retrocausal theories

Now let's see retrocausal theories. It has been argued that these theories can provide a local, hidden-variables explanation of the Bell inequality-violating correlations predicted by quantum mechanics [10-12]. The key assumption is that in a Bell test experiment the measurement settings as a cause (in the future) can affect the hidden-variable distribution during preparation (in the past) by a retrocausal mechanism. For example, in the EPR-Bohm experiment discussed before, Alice's measurement setting a and Bob's measurement setting b may retrocausally affect the values of spin of the particles 1 and 2 along these directions (as hidden variables of the theory) during preparation, so that these values are correlated in a way to be able to explain the correlations between Alice's and Bob's measurement results which are assumed to directly reflect these values of spin.

It is well known that Bell's theorem prohibits that the spin of the particles have definte values in all directions during preparation in a local hidden-variables theory. This is not a problem for a retrocausal theory to explain the experiment, since during each run of experiment there are only two angles a and b that can be set to measure the particle pair, and it is only necessary for a retrocausal theory to ensure that the spin of the particles have definite values along these two directions and these values are correlated [10]. However, in a Gedankenexperiment with superobservers, since Alice's and Bob's measurements can both be undone locally, a Bell test experiment can be repeatedly made on the same particle pair with all possible angles of a and b. Thus, a retrocausal theory will need to ensure that the spin of the particles have definite values along all directions. But this contradicts Bell's theorem.

In addition, it seems that a retrocausal theory cannot explain the stronger correlation between the results of Alice's measurements and Bob's measurement choice in the previous Gedankenexperiments either. Consider the case that Bob first measures the spin of particle 2 at an arbitrary angle b, and then Alice measures the spin of particle 1 at the same angle b and obtains her result. A retrocausal theory can explain the perfect anti-correlation between the two results by a retrocausal mechanism. For example, Alice's and Bob's measurement settings retrocausally affects the values of spin of the particles 1 and 2 along the direction b during preparation so that these values are anti-correlated. Then Alice's and Bob's spin measurements directly reflect these values of spin, and thus their results are also anti-correlated.

But a problem appears when the superobserver undoes Alice's measurement and then Alice measures again the spin of particle 1 at angle b and obtains her second result. Since the superobserver's reset operation is local, it does not affect either particle 2 and Bob or the preparation source retrocausally. Moreover, since the superobserver's reset operation is not necessarily an exact time-reversal, and it only needs to restore the wave function of relevant systems to their initial state, it will not restore the values of all hidden variables such as the spin of particle 1 to their initial values in general. Thus, when Alice measures again the spin of particle 1 at angle b, her second result may be different from her first result and thus will be not anti-correlated with Bob's result. This contradicts the predictions of quantum mechanics.

5.4 Bohm's theory

If a local theory such as superdeterminism and a retrocausal theory cannot explain the correlation, then the correlation can only be explained by nonlocal processes or actions at a distance. A well-known example of such nonlocal theories is Bohm's theory [34].

According to Bohm's theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The Bohmian laws of motion are expressed by two equations: a guiding equation for the configuration of particles and the usual Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. A major feature of Bohm's theory relevant to my following analysis is that the theory is manifestly nonlocal. According to the guiding equation, the velocity of any particle of a many-particle system depends on the positions of the other particles when the wave function of the system is entangled, no matter how far apart these particles are.

Consider the case that Bob first measures the spin of particle 2 at an arbitrary angle b, and then Alice measures the spin of particle 1 at the same angle b and obtains her result. According to the Bohmian laws of motion, immediately after Bob's measurement, particle 2 will stay in one branch of the spin singlet state corrresponding to the measurement result, such as the b-spin up branch corrresponding to the b-spin up result. Then, according to the guiding equation, the motion of particle 2 will be determined only by its b-spin up wave function, and correspondingly, the motion of particle 1 will be determined only by its b-spin down wave function, not by the whole spin singlet state. Here there is a genuine action at a distance; Bob's measurement instantaneously influences particle 1 in Alice's side. Then, when Alice measures the spin of particle 1 at angle b, her result must be b-spin up, which is perfectly anti-correlated with Bob's result. Moreover, when the superobserver undoes Alice's measurement and Alice measures again the spin of particle 1 at angle b, she will also obtain the same b-spin up result. This is because according to the guiding equation, the motion of particle 1 nonlocally depends on the position of particle 2, and since particle 2 stays in its b-spin up wave function, the motion of particle 1 is still determined by its b-spin down wave function.

On the other hand, when Alice and the superobserver first make their series of measurements and reset operations and then Bob makes his measurement, Bob's measurement does not influence the results of Alice's measurements, which are determined only by the initial position of particle 1 according to the guiding equation. Since the initial position of particle 1 satisfies the Born rule, Alice will obtain two different results, spin up and spin down, with roughly equal frequency. In this way, Bohm's theory can explain the stronger correlation between the results of Alice's measurements and Bob's measurement choice.

It has been shown that in Bohm's theory, the joint distributions given by the Born rule for position measurements cannot in general agree with the distributions of the actual Bohmian particle positions in all Lorentz frames [35]. This is not beyond expectations due to the existence of action at a distance in the theory. Moreover, a similar result has been obtained in other hidden-variable theories such as the modal interpretation [36]; it is shown that in these theories special relativity is violated and a preferred Lorentz frame exists at the assumed ontological level. The incompatibility proof given previously further shows that a preferred Lorentz frame must also exist in hidden-variable theories when considering only actually observed measurement results (see also [33]).

It is worth noting that in deterministic hidden-variable theories such as Bohm's theory, when the superobserver's reset operation is an exact time-reversal, it will also restore the values of all hidden variables such as the positions of all Bohmian particles to their initial values. Then, in the previous Gedankenexperiments, the results of Alice's measurements will be all the same, independently of Bob's measurement[11], and thus the incompatibility proof cannot go through. As noted before, however, the superobserver's reset operation is not necessarily an exact time-reversal in general, and it only needs to restore the wave function of Alice and the spin wave function of the particles to their initial states. Therefore, the incompatibility proof still applies to deterministic hidden-variable theories.[12]

5.5 Other unitary quantum theories

How about other unitary quantum theories? As I have argued above, if a unitary quantum theory assumes that measurement results are unique and not frame-dependent, then the incompatibility proof will directly apply to the theory.

Here an interesting issue may appear, which is to determine whether a quantum theory is a unitary quantum theory. A unitary quantum theory is composed of the universal Schrödinger equation and the Born rule. The universality of the linear Schrödinger equation requires that the wave function never collapses (either ontologically or epistemically), and thus permits that a measurement such as Alice's measurement can be undone by a superobserver in principle (i.e. the wave function of the measured system and the measuring device after a measurement can be reset to the initial one). The Born rule says that the modulus squared of the wave function gives the probabilities of possible measurement results. A unitary quantum theory thus defined may be regarded as the core of quantum mechanics, and it does not depend on any particular interpretation of the theory, such as the ontological status and meaning of the wave function and how to solve the measurement problem.

Take the non-ψ-ontic quantum theories as an example. These theories

[11] In this case, when Bob makes his measurement at last, the statistics of Alice's results will violate the Born rule, and the violation may exist in all Lorentz frames. Although this is understandable in a deterministic hidden-variable theory and does not contradict existing experience either, it seems to suggest that an indeterministic hidden-variable theory is more consistent with quantum mechanics [37-39].

[12] Interestingly, an analysis of the invariance of the results of Alice's and Bob's spacelike separated measurements in different Lorentz frames also shows that deterministic (nonlocal) hidden-variable theories are incompatible with special relativity and require the existence of a preferred Lorentz frame [40].

may include consistent histories [41], ψ-epistemic models [42], pragmatist approaches to quantum mechanics [43], and QBism [44]. In these theories, although the wave function is not real and thus the collapse of the wave function is not a real physical process, the wave function may collapse at the epistemic level or as part of a mathematical rule. Thus it seems debatable whether these theories belong to unitary quantum theories as defined above. However, in the previous Gedankenexperiments, if only Alice's measurement does not influence Bob's side instantaneously or faster than light, it seems that we can still assign the same initial wave function to Alice, the particles and Bob after the superobserver's reset operation. If any of the above theories accepts this, then the incompatibility proof will apply to it. In other words, the theory will be also incompatible with special relativity.

6 Many worlds

Finally, let's see the many-worlds interpretation of quantum mechanics (MWI). It is widely thought that MWI evades Bell's theorem by rejecting unique outcomes, and it is consistent with special relativity. Then, does the incompatibility result also hold true in MWI? I think the answer is positive.

Consider an EPR-Bohm experiment in which Bob first measures the spin of particle 2 at an arbitrary angle b, and then Alice measures the spin of particle 1 at the same angle b and obtains her result. According to Wallace [45], from the point of view of another non-interacting observer, the state of an observer after her measurement is indefinite, and the state of the region is "a nonclassical state instantiating two sets of observers with macroscopically different observations" [45, p.308]. But in the EPR-Bohm experiment, after Bob first measures the spin of particle 2 at an angle b and then Alice measures the spin of particle 1 at the same angle, the particle and observer in each region already have definite spin and result states relative to definite spin and result states of the particle and observer in the other region. In the words of Brown and Timpson [9], "following the two local measurements of Alice and Bob, from the point of view of one side, the states of the systems in the other side already correspond to a definite, perfectly anti-correlated, measurement result." For example, from the point of view of each version of Bob, such as $B+$ who obtains the result spin-up, there is a unique version of Alice who already obtains a definite, perfectly anti-correlated, measurement result, such as $A-$ who obtains the result spin-down. In the anti-correlation case or parallel case, unlike the non-parallel cases,[13] it is not required that a joint measurement should be performed,

[13] According to Brown and Timpson [9], in the non-parallel cases, we can only think of the correlations between measurement results on the two sides of the experiment actually obtaining in the overlap of the future light-cones of the measurement events, and thus there will be no violation of special relativity [9].

comparing the results from Alice and Bob, which can only take place in the overlap of the future light cones of the measurements of Alice and Bob [9].

This understanding of MWI is arguably proper. In MWI, although measurement results are not unique, they are objective after all. For example, after Bob performs his spin measurement, although there are two versions of Bob, each version such as $B+$ already obtains a definite result such as spin-up. Similarly, after Alice performs her spin measurement, although there are two versions of Alice, each version such as $A-$ already obtains a definite result such as spin-down. Then, in the above EPR-Bohm experiment, since the entangled state of the composite system including Alice and Bob is nonlocal in MWI, and the nonlocal property of the composite system ensures the perfect anti-correlation between the measurement results of corresponding versions of Alice and Bob, the perfect anti-correlation should exist immediately following their local measurements, not only after they meet or a joint measurement is performed to compare their results.

If the perfect anti-correlation does not exist immediately following the two local measurements of Alice and Bob, then when will it appear? When a joint measurement is performed to compare the results, the perfect anti-correlation will be confirmed. But it is obvious that the perfect anti-correlation is not generated by the joint measurement; if Alice and Bob do not measure the spin of particles 1 and 2 at the same angle, then there will be no necessarily perfect anti-correlation between their results. In addition, the spreading out of the branching process from the local measurements of Alice and Bob does not change the measurement result of each version of Alice or Bob, and thus does not change the correlation between these results either. Therefore, it is arguable that the perfect anti-correlation exists immediately following the two local measurements of Alice and Bob.

This result does not cause problems in the usual EPR-Bohm experiment. However, in the variant of the EPR-Bohm experiment with a superobserver, it does cause a problem or a contradiction. In a Lorentz frame where Alice's measurement is undone after Bob's measurement, from the point of view of each version of Bob, such as $B+$ who obtains the result spin-up, there is a unique version of Alice who obtains a definite, perfectly anti-correlated, measurement result, such as $A-$ who obtains the result spin-down. But in the laboratory frame where Alice's measurement is undone before Bob's measurement, from the point of view of each version of Bob, such as $B+$ who obtains the result spin-up, there is *no* unique version of Alice who obtains a definite, perfectly anti-correlated, measurement result, such as $A-$ who obtains the result spin-down. This is similar to the argument in the single-world theories (see Section 4).

Also like the single-world theories, the contradiction is more obvious when considering that Alice's and Bob's measurement operations are independent of each other when they are spacelike separated in MWI. In this case, when Alice's measurement is undone before Bob's measurement, Alice's and Bob's measurements are equivalent to two independent measurements on two spin singlet states, and thus their measurement results will be independent of each other. This means that in the laboratory frame (where Alice's measurement is undone before Bob's measurement), from the point of view of each version of Bob, there are two versions of Alice, one of which obtains a definite, perfectly anti-correlated measurement result, while the other of which obtains a definite, perfectly correlated measurement result, and the probability of each version is the same 1/2. But in another Lorentz frame where Alice's measurement is undone after Bob's measurement, from the point of view of each version of Bob, there is a unique version of Alice who obtains a definite, perfectly anti-correlated, measurement result. Thus the contradiction will appear with probability 1/2.

7 Conclusions

In this paper, I analyze the important issue of whether unitary quantum theories are compatible with special relativity. I propose a new Gedankenexperiment, a variant of the EPR-Bohm experiment with a superobserver who can undo a measurement. In this Gedankenexperiment, there is a stronger correlation (between the results of two spacelike separated measurements) than the correlation investigated in Bell's theorem. Based on an analysis of the correlations in different Lorentz frames, I prove that unitary single-world theories, including retrocausal theories and superdeterminism, are incompatible with special relativity, and in order to avoid the incompatibility, there must exist a preferred Lorentz frame in these theories. Moreover, I argue that the incompatibility proof also applies to a proper version of the many-worlds interpretation of quantum mechanics. This closes the major theoretical loopholes of Bell's theorem. Finally, I also analyze the implications of this new result for unitary quantum theories. It is argued that the stronger correlation found in the Gedankenexperiment cannot be explained by retrocausal processes or even the common causes in the past, but only be explained by nonlocal processes or actions at a distance.

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